Random Material's Characteristics to Study Fluid-Structure Interaction

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Summary: In many situations, the vibrating structures are in contact with a fluid (fluid around the hulls of a boat, reservoirs, heat exchangers in power plants, etc.), but the dynamic behavior of the structure can be significantly modified by the presence of the fluid. The sizing must take into account the effects of fluid-structure interaction. Traditionally, the study of mechanical systems fluid-structure interaction is based on a deterministic approach where all the parameters used in the model are a fixed value. But it suffices to having conducted a few experimentations to realize that the limitations of such modeling. Hence it needs to take into accounts the uncertainty in the parameters of mechanical systems. This work proposes to take the characteristics of the structure and the fluid as random and shows the efficiency of such approach. The proposed numerical stochastic method of the modal synthesis extended to reliability study, based on FORM (First Order Reliability Method) and SORM (Second Order Reliability Method) approaches, for solving the large vibro-acoustic problems. The numerical method used takes into account the uncertainties of the input parameters of the two domains. The application of the proposed method is performed on a boat propeller immersed in air and water. To validate the calculation process, the numerical study is compared to an experimental study.

Keywords: fluid-structure interaction; modal synthesis; numerical simulation; random material characteristics; reliability; vibro-acoustic

Introduction

The vibrating structures are often in contact with a fluid (fluid around the hulls of a boats, reservoirs, heat exchangers in power plants, etc.), but the dynamic behavior of the structure can be significantly modified by the presence of the fluid. The sizing must take into account the effects of fluid-structure interaction. Particularly the dynamic behavior of an elastic structure coupled to a fluid can be significantly modified through the presence of the fluid.

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In the vibro-acoustic studies of coupled systems fluid-structure modeled by the finite element method,[1–4] the interest of reducing the size of the problem is obvious because we have to add all the degrees of freedom of the acoustic domain to those of the structure.

Traditionally, the study of mechanical systems fluid-structure interaction based on a deterministic approach where all the parameters used in the model are a fixed value. But it suffices to having conducted a few experimentations to realize that the limitations of such modeling.[5–7] Hence the need to take into accounts the uncertainty on the parameters of mechanical systems. Furthermore the knowledge of the variation of the response of a structure involving the uncertain parameters, geometry, boundary conditions, manufacturing tolerances and loads is essential in the design process. It is indeed widely recognized that the small uncertainties in these parameters can have a significant influence on the vibration behavior of the studied system. However, to work on realistic modeling uncertainties integration is essential.

This work proposes to take the characteristics of the structure and the fluid as random and shows the efficiency of such approach. The used numerical stochastic method of the modal synthesis extended to reliability study, based on FORM and SORM approaches, for solving the large vibro-acoustic problems. The method developed Couples the dynamic substructuring method of type Craig and Bampton and the acoustic subdomains method based on a pressure formulation.[8] The numerical method used takes into account the uncertainties of the input parameters of the two domains.[9] The application of the proposed method is performed on a boat propeller immersed in air and water. The numerical study is conducted using a code developed in MATLAB© coupled with the finite element code ANSYS© in order to evaluate the reliability of the structure. This numerical study compared to an experimental study we can validate the calculation process and the method proposed in the field of frequency analysis and reliability-study of submerged structures to build a reliable and robust model for the problems of fluid-structure interaction.

Problem Statement

Front of the complexity of mechanical systems fluid-structure interaction, the classical numerical and experimental methods of the vibratory mechanical are costly, sometimes even unusable. In perfect coherence with the modular organization of large projects, the method of substructure and subdomain appears to be the most effective way to conduct the vibratory study for all fields from dynamic domains constituting the system.[10] However, the application of a modal synthesis method for the vibro-acoustic problem raises two crucial problems linked on the one hand to the choice of acoustic formulation and secondly the choice of method associated subdomains.[11–13] The respective merits of this approach will be compared and illustrated by an example of implementation on an industrial problem of fluid-structure interaction in vibro-acoustic. We consider an acoustic fluid in a rigid cavity, in contact with an elastic structure. The variables used to describe the structure Ω and the acoustic cavity F are respectively the displacement u and the pressure fluctuation p. We note ρ_f the fluid density, C the speed of sound in the fluid, and ρ_s, E and ν are respectively the density, Young's modulus and Poisson's ratio of the structure.

In the sequel, the exhibitors (and indexes) s and f shall designate respectively the numbers of substructures and subdomains.

Each substructure occupies a volume noted Ω^s. Each subdomain fluid occupies a volume Ω^f. The structure Ω is composed of N^s substructures Ω^s ($s=1,\ldots,N^s$) and the acoustic fluid F is constituted by N^f acoustic subdomains Ω^f ($f=1,\ldots,N^f$). Into we distinguish three types of interface, defined as follows:

$$L^{ss'} = \Omega^s \cap \Omega^{s'},\ L^{ff'} = \Omega^f \cap \Omega^{f'},$$
$$L^{sf} = \Omega^s \cap \Omega^f$$

$L^{ss'}$ denotes the interface (or junction) between the substructure Ω^s and the substructure $\Omega^{s'}$ ($L^{ss'}=\emptyset$ if these two domains are not in contact).

$L^{ff'}$ represents the interface between the fluids subdomains Ω^f and $\Omega^{f'}$ ($L^{ff'}=\emptyset$ in the absence of contact between the two subdomains fluid).

$L^{sf'}$ is the fluid-structure interface between the substructure Ω^s and the fluid subdomain Ω^f ($L^{ff'}=\emptyset$ if Ω^s and Ω^f are not in contact).

The finite element discretization of variational formulations structural and acoustic leads to the following algebraic form:

$$(K^s - \omega^2 M^s)\{u^s\} = \{F_e^s\} + \sum_{\substack{s'=1\\ s'\neq s}}^{N^{Ss}} \left\{F_L^{ss'}\right\} s = 1, \ldots, N^s \tag{1}$$

where:

$\{u^s\}$ the displacement vector of each substructure,

$[M^s]$ the mass matrix of the substructure Ω^s,

$[K^s]$ the stiffness matrix of the substructure Ω^s,

$F_L^{SS'}$ the bonding forces,

F_e^S the equivalent external forces including all excitations of type imposed displacement.

For subdomains

$$\left(H^f - \omega^2 E^f\right)\{P^f\} = \{a_e^f\} + \sum_{\substack{f'=1 \\ f' \neq f}}^{N^f} \left\{a_L^{ff'}\right\} \quad f = 1, \ldots, N^f \tag{2}$$

where:

$\{p^f\}$ the acoustic pressure vector of each subdomain,

$[H^f]$ the mass matrix of the fluid subdomain Ω^f,

$[E^f]$ the stiffness matrix of the fluid subdomain Ω^f,

$\{a^f\}$ the vector of equivalent external pressures (including exceptions inherent in frontier Γ_p^f of type imposed pressure),

$\{a_L^{ff'}\}$ the acceleration at the interface between the fluid subdomain Ω^f and the fluid subdomain $\Omega^{f'}$.

For vibro-acoustic problem

The structural and fluids degrees of freedom are grouped in a global vector:

$$\langle up \rangle = \left\langle u^1 u^2 \ldots u^{N^s} p^1 p^2 \ldots p^{N^f} \right\rangle$$

Taking into account the interaction between the substructure Ω^s and the fluid subdomain Ω^f, the global matrix of fluid-structure interaction is written:

$$[L] = \begin{bmatrix} L^{11} & \cdots & L^{1N^f} \\ \vdots & \ddots & \vdots \\ L^{N^s 1} & \cdots & L^{N^s N^f} \end{bmatrix} \tag{3}$$

In this expression, $[L^{sf}]$ are implicitly zero when there is no interface between the sub-structure Ω^s and the fluid subdomain Ω^f.

Equation (4) can be assembled into a single equation (5):

$$\{u\} = [S][\varphi]\{w\},\ \{p\} = [T][\psi]\{r\} \tag{4}$$

$$\begin{Bmatrix} u \\ p \end{Bmatrix} = [C][R]\begin{Bmatrix} w \\ r \end{Bmatrix} \tag{5}$$

where: $[C] = \begin{bmatrix} S & 0 \\ 0 & T \end{bmatrix}$ and $[R] = \begin{bmatrix} \varphi & 0 \\ 0 & \psi \end{bmatrix}$

Taking into account the compatibility conditions at the interface the last terms of equations (1 and 2) will disappear. Finally, the equation to solve of vibro-acoustic problem reduced by modal synthesis without the external loads is:

$$[C]^t[R]^t\left(-\omega^2\begin{bmatrix} M & 0 \\ \rho_f L^t & E \end{bmatrix} + \begin{bmatrix} K & -L \\ 0 & H \end{bmatrix}\right) \times [C][R]\begin{Bmatrix} w \\ r \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{6}$$

Reliability Analysis

The first step in the analysis of the reliability is to define the design variables X_i $(i = 1, 2, \ldots, n)$ having a significant level of fluctuation (n being the number of random variables).[14–17] For each of these variables X_i (whose achievements are noted x_i), we assign a probability distribution reflecting the corresponding random. This can be achieved through statistical studies, physical findings, or lack of resources, expert advice. The quality of information is reflected in the accuracy of results. The second step is to define a number of potential failure scenarios. For each of them, a performance function G (x_i) divides the space into two regions of the variables: safety domain G $(x_i) > 0$ and failure domain G $(x_i) \leq 0$. The boundary between these two domains is defined by G $(x) = 0$, called limit state function. The failure probability is given by:

$$p_f = \int_{G \leq 0} f_{X_1,\ldots,X_n}(x_1, \ldots, x_n)dx_1 \ldots dx_n \tag{7}$$

where $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)$ is the joint density of probability of the variables Xi. The evaluation of this integral is very costly in time of calculation, because it comes to a very small quantity and because all the necessary information on the joint density of probability is not available. And it is very rare that this integral can be studied analytically or numerically. In practice, we do not have in general the joint density of probability of the vector {X}, we must be content most often the marginal distributions for each variable Xi and some information about their correlation. In addition, the integration domain can be defined implicitly and complex according the used mechanical model. Various methods of resolution have been developed to overcome these difficulties.[5,6,18,19]

Traditionally, we distinguish two main methods: the methods based on simulations such as Monte Carlo simulation and those using approximate methods such as FORM and SORM, these two approximation methods are based on the determination of the reliability index β, which allows to access an approximate value of probability of failure Pf, and they are illustrated in figure 3 around the design point.[20]

For most of the methods of calculation, the calculation of the probability of failure is to be placed, not in the space of physical variables, but in a standard space. The vector of basic variables X, defined by its joint density function is transformed into a vector of centered normal variables, normalized and independent U, defined in the standard space.[21] This transformation consists in particular to pass of random variables xi of arbitrary marginal distributions and optionally correlated to reduced and centered normal distribution uncorrelated (independent) u_i, by the following expression:

$$u_i = T_i(x_k) = \Gamma_{ij}\phi^{-1}\left[F_{X_j}(x_j)\right] \tag{8}$$

where Γ_{ij} is the inverse of the Choleski triangularization of the correlation matrix equivalent, $F_{X_j}(x_j)$ is the distribution function of the variable x_j, ϕ^{-1} is the inverse of the law standard Gaussian distribution.

In standard space, the reliability index β is the minimum distance between the origin and the limit state function. This distance defines a point P*, said the design point (or point of failure most likely). The index β is calculated by solving an optimization problem with constraints:

$$\beta = \min\sqrt{\sum_i T_i^2(x_j)}\ \text{sous}\ G(x_j) \le 0 \tag{9}$$

A calculation method of the transformation is available in,[22] where details are given on an approximate computation of the equivalent matrix of correlation.

Numerical Results

Following our deterministic study applied to a boat propeller and a single blade in air and in water, we have noticed a variation of the numerical results compared to the experimental results, we have thought to extend this study to the stochastic and reliability study to take account of uncertainties in the entered variables.

Figure 1 shows the finite element model of the structure. The fluid and the propeller are defined by their properties shown in the Tables (1) and (2). To check the reliability of this structure the first natural frequency R1 is analyzed.

The objective of this study is the demonstration of the interest of the proposed method. The numerical development has been realized by a code which couples MATLAB and ANSYS. The learning methods are validated against different criteria. The value of the failure's probability and the calculation of the reliability index. The method of reduction is applied to a given simplified model of the propeller composed of four substructures and the acoustic cavity is divided into four subdomains containing each approximately the same number of elements see Figure 2. The deterministic numerical calculations are performed on the whole structure and on the single blade and they are compared with experimental results.[9–23]

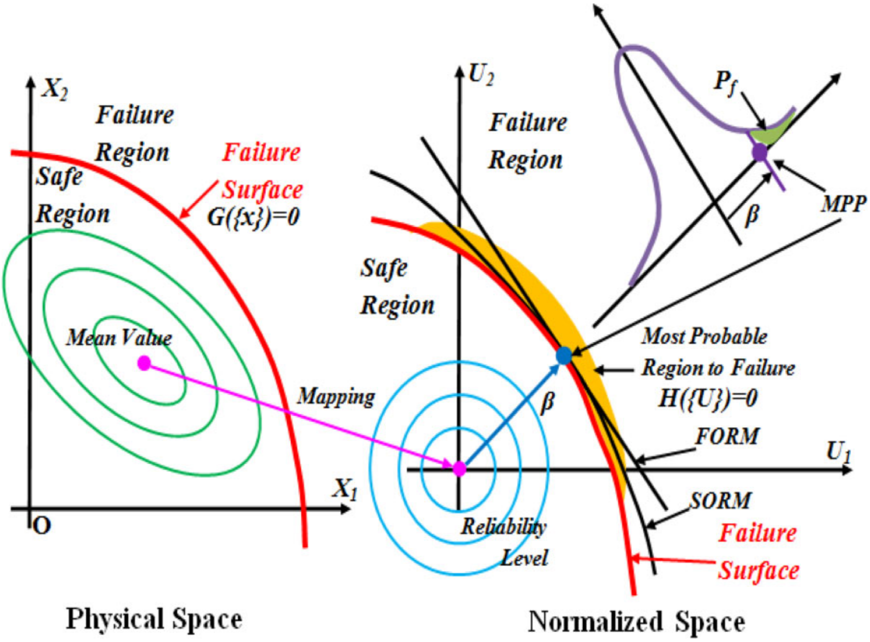

Figure 1.
Approximation of the limit state near the point of failure most probable.

Probabilistic Study

The highlighting of the important dispersion of material properties of vibro-acoustic problems has incited us to turn towards stochastic methods for their analysis. Front of the complexity of the problem, we have chosen to consider in this work only the sources of uncertainties related to the material properties and we will be limited to the study of a single blade in air and in water. But the uncertainties regarding the other elements of the structure (geometry, boundary conditions and mechanical behavior) have not been taken into account in a perspective of simplification.

The choice of standard deviations and the means of random variables were chosen based on deterministic and experimental analyzes.[23,24] The considered standard deviations were also adjusted to maintain realistic ranges of materials involved. The table (3) contains the means of random variables and their standard deviations used in this study and the distributions laws chosen.

Table 1.
Material properties of the structure.

Young's modulus Pa	Poisson's ratio	Density Kg.m-3
9.6×10^{10}	0.3	9200

Table 2.
Material properties of the fluid.

Density Kg.m-3	Speed of sound m.s-1
1000	1500

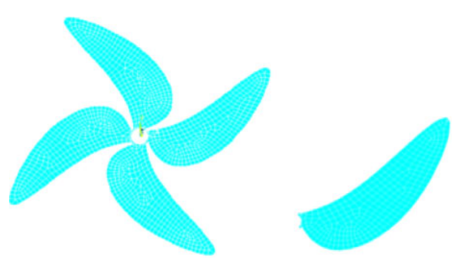

Figure 2.
Finite element model.

Table 3.
Moments of the parameters of the problem and distribution laws.

Parameters	Young's modulus Pa	Density of structure Kg.m-3	Density of the fluid Kg.m-3
Distribution	Gaussian	Uniform	Uniform
Means	9.6×10^{10}	9200	1000
Standard deviation	0.5×10^{10}	2669.6	295.72

In this context the stochastic calculation was carried out using probabilistic design system of the ANSYS. This tool is based on a calculation with Monte Carlo simulation (MC) for 100 samples and the response surface method (RSM) for 40 samples. Tables 4 and 5 show means and standard deviations of the natural frequencies. The first gives the finding results using the propeller blade in air and the second gives the finding results using the propeller blade in water.

Reliability Study

The finite element code ANSYS present the probabilistic modules but it does not allow access to the source files, which is a major handicap in the perspective of implementation of the coupled model. To overcome this difficulty, we have chosen to implement a direct coupling between a reliability code developed in MATLAB and ANSYS. These two codes in fact answer our needs in terms of calculation capacity and the possibility of dialogue. The first step consists of the declaration in the MATLAB code of the random variables of the model (laws of distribution and associated parameters), the failure function G and all the necessary parameters to solve the reliability calculation. From this information, this probabilistic code can generate to achievements of vector {X} of random variables. In the second step we appeal to ANSYS to deduce through the finite element calculation the first frequencies of the propeller induced in the different elements of the structure for the draw {X}. MATLAB code shall then have all the information to assess the quantity G ({X}). These different steps are repeated until convergence of the optimization algorithm for obtaining the reliability index and the probability of failure.

In this numerical study, the analysis of the reliability of the propeller in air and in water was based on an implicit limit state function G based on the first natural frequency R_1.

- For the propeller in air:

$$G(E, \rho_s) = R_1 - R_0, \text{ with } R_0 = 73\text{Hz} \quad (10)$$

- For the propeller in water:

$$G(E, \rho_s, \rho_f) = R_1 - R_0, \text{ with } R_0 = 36 \quad (11)$$

The mean values of random variables and their standard deviations as well as

Table 4.
Means and standard deviations of the natural frequencies for the propeller in air.

Modes	R1 Hz	R2 Hz	R3 Hz
ANSYS	74.863	119.82	205.58
EXPERIMENTAL	73	117	201
MC	72.46	115.8	199.6
RSM	73.58	118.5	194.7
SD	12.5	23.5	37.1

Table 5.
Means and standard deviations of the natural frequencies for the propeller in water.

Modes	R1 Hz	R2 Hz	R3 Hz
ANSYS	37.71	67.54	126.32
EXPERIMENTAL	36	65	123
MC	35.671	66.34	124.12
RSM	34.22	67. 45	121.93
SD	5.62	11.16	25.871

Table 6.
Design parameters and their statistical moments considered in the propeller in air.

Parameters	Young's modulus Pa	Density of the structure $Kg.m^{-3}$	reliability index β	Probability P_f
FORM	8.5×10^{10}	9030	3.68	0.083
SORM	8.5×10^{10}	9030	3.68	0.012

Table 7.
Design parameters and their statistical moments considered in the propeller in water.

Parameters	Young's modulus Pa	D. of the structure $Kg.m^{-3}$	D. of the fluid $Kg.m^{-3}$	β	P_f
FORM	8.37×10^{10}	8980	890	3.54	0.11
SORM	8.37×10^{10}	8980	890	3.54	0.087

distributions chosen for this study are shown in Table (3).

Tables (6) and (7) summarize the design parameters and their statistical moments considered in the uncoupled and coupled structure for this example, and they illustrate a comparison between the results obtained from FORM and SORM approaches.

Discussion

In the deterministic case the numerical results give a little far from those obtained by experimentation, with an uncertainty compared to that given by other authors.[23] To overcome this problem we extend our study to a stochastic study (Tables 4 and 5) which consists, firstly, to implement a simulation technique based on the Monte Carlo method and response surface method, then secondly, to make a reliability analysis. This technique involves a particular treatment of inputs and outputs random variables in order to build a trusted domain on the parameters of the studied system.

Table 8.
Natural frequencies for the propeller in air calculated by FORM and SORM.

Modes	R1 Hz	R2 Hz	R3 Hz
FORM	70.35	116.44	201.24
SD	3.44	5.78	7.39
SORM	69.87	115.63	201.09
SD	3.18	5.36	7.21

Table 9.
Natural frequencies for the propeller in water calculated by FORM and SORM.

Modes	R1 Hz	R2 Hz	R3 Hz
FORM	36.56	65.73	123.49
SD	1.48	3.65	4.18
SORM	35.67	65.58	123.24
SD	1.56	3.83	4.21

On the basis of preliminary deterministic study, the reliability analysis based on FORM and SORM was conducted for the blade in air and in water. Precisely given the low values of probability of failure P_f, it seemed to us more convenient to reasoning in terms of reliability index β, in order to build a trust domain of the input parameters chosen as defined in Table (3).

By comparing the probabilities of failure and reliability index calculated and displayed in Tables (6) and (7) with the ranges of values of probability of failure and reliability index β corresponding accepted in various industrial sectors in particular for the marine structures (($P_f \in [10^{-2},10^{-4}]$ and $\beta \in [2.33,3.72]$)), we find that there is a very important level of reliability of the blade. A prior study of the sensitivity of material parameters was performed to identify the dominant parameters at the behavior of materials. By a more rational treatment of uncertainties, the reliability approach allows a better appreciation of the safety margins with the aid of the objectives indicators of confidence, and in this sense is an appropriate tool to help to the decision in phases of design and maintenance.

Conclusion

This work proposes a probabilistic numerical method of modal synthesis extended to reliability study based on FORM and SORM approaches for solving large size vibro-acoustic problem of coupled fluid-structure systems modeled by the finite element method. The developed method couples the dynamic substructuring method of type Craig and Bampton and acoustic subdomains method based on the acoustic pressure formulation. And to take into account uncertainties related to parameters of the two domains, a reliability analysis was subsequently conducted. For this purpose an integrated approach combining the methods of reliability and finite element modeling has been proposed to account for the failure of submerged structures. From the point of sight of designers, this approach provides an adequate framework for the analysis of the reliability of structures in contact with the fluid which confers a physical significance to the uncertainties introduced.

The used numerical method takes into account the uncertainties of input parameters such as properties of the two domains fluid and solid. The application of the proposed method is performed on a propeller boat in air and water. The numerical study is performed using a code developed which couples MATLAB© and ANSYS© to evaluate the reliability of the structure. The comparison of the numerical results allows us to validate jointly the process of calculation and the method proposed in the domain of frequency analysis and the reliability of submerged structures in order to build a reliable and robust model for the problems of fluid-structure interaction.

The obtained results through the study of the propellers are very encouraging. The proposed model, whose choices have been dictated by the physical phenomena involved, the deterministic results and available experimental data. This model seems indeed capable to account for the reliability of these submerged structures at different scales. If the deterministic study has shed light on the effect of coupling between the fluid and the structure, the stochastic study has demonstrated the relevance of this approach with a view to improve the robustness of the forecast results in the probabilistic approach.

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